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Mach's Principle and Model for a Broken Symmetric Theory of Gravity

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We investigate spontaneous symmetry breaking in a conformally invariant gravitational model. In particular, we use a conformally invariant scalar tensor theory as the vacuum sector of a gravitational model to examine the idea that gravitational coupling may be the result of a spontaneous symmetry breaking. In this model matter is taken to be coupled with a metric which is different but conformally related to the metric appearing explicitly in the vacuum sector. We show that after the spontaneous symmetry breaking the resulting theory is consistent with Mach's principle in the sense that inertial masses of particles have variable configurations in a cosmological context. Moreover, our analysis allows to construct a mechanism in which the resulting large vacuum energy density relaxes during evolution of the universe.

KEY WORDS:

1. INTRODUCTION

One of the interesting possibilities concerning the origin of gravitational coupling is that it may be the result of an invariance breaking of some fundamental symmetry of nature. In recognition of such a symmetry, the first possibility may be that like other fundamental interactions, the coupling constant of gravitational interaction has its origin in a spontaneous symmetry breaking at some appropriately energy scales. In fact, Zee (1979) has employed a scalar tensor theory to show that a spontaneous symmetry breakdown at Planck scale would lead to a gravitational coupling as suggested by general relativity. In this approach, one recognizes two problems: firstly, the model seems not to be consistent with Mach's principle in the sense that after the symmetry breaking the model reduces to general relativity and specifically the gravitational coupling is given by the gravitational

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constant.[2] The implication is that scalar tensor theories motivated by Mach's principle (Brans and Dicke, 1961) have no relevance at energy scales lower than the Planck scale. Secondly, it is well known that the proposed symmetry breaking leads to the appearance of a vacuum energy density which is enormously larger than the experimental upper limit (Weinberg, 1989).

There is another possibility about the kind of symmetry which may be of significance. Since gravitational coupling is a dimensional coupling, its strength can be changed by a unit transformation. Thus, the corresponding symmetry which is expected to have important role is conformal symmetry. One may study an invariance breaking effect in a conformally invariant gravitational model by introducing a constant mass scale. It has been shown (Deser, 1970; Salehi, 1998; Salehi, and Bisabr, 2000) that such an invariance breaking also leads to a gravitational coupling with the same strength as used in general relativity.

Our main purpose in the present work is to keep the idea that the gravitational coupling arises from an invariance breaking effect with special concern for addressing the two aforementioned problems. We shall study spontaneous symmetry breaking in a conformally invariant scalar tensor theory in which the scalar field has a quartic self-interaction term. The basic ingredient in the theory is that we take the two metric tensors describing the gravitational and the matter parts to belong to different conformal frames. We then consider the conformal factor relating the two metric tensors as a dynamical field. Such a dynamical field is basically imposed in our model to make a dynamical distinction between the two unit systems usually used in cosmology and elementary particle physics. We have already emphasized the significant role of this dynamical distinction in construction of a mechanism to reduce a large cosmological constant during evolution of the universe (Bisabr and Salehi, 2002). In the present work, we intend to investigate the role of conformal symmetry in a scalar tensor theory that undergoes spontaneous symmetry breaking. We argue that if the theory is taken to be conformally invariant, after the spontaneous symmetry breaking it remains consistent with the spirit of Mach's principle. We also show that the resulting vacuum energy density appears as a decaying cosmological constant.

We organize this paper as follows: In Section 2, we first offer a brief review of the model proposed by Zee (1979). In Section 3, we use a gravitational model which is conformally invariant. We argue that there is an ambiguity concerning the coupling of matter systems to such a model. In general, matter systems should be coupled to the metric which is conformally related to that describing the vacuum sector. In Section 4, we consider spontaneous symmetry breaking in this gravitational model. We discuss the problem of the emergence of a large vacuum

[2] It should be remarked that Mach's idea on the nature of inertia has found a limited expression in general relativity. For a detailed discussion see, for example, Brans and Dicke (1961).

energy density and consistency of the model with Mach's principle. In Section 5, we outline our results.

Throughout the following we shall use units in which $\hbar = c = 1$ and the signature is $(- + ++)$.

2. A BRIEF REVIEW OF ZEE'S MODEL

Spontaneous symmetry breaking is one of the key ideas in elementary particle physics which is expected to lead to unification of strong, weak and electromagnetic interactions. In order to incorporate this mechanism into gravity, Zee proposed (1979) a modification of the Einstein–Hilbert action used in general relativity. The proposed model is

$$S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}\phi^2 R + \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right\} + S_m(g_{\mu\nu}, \phi), \tag{1}$$

where ϕ is a scalar field with a potential $V(\phi)$, R is the curvature scalar, ∇_μ denotes covariant differentiation, and $S_m(g_{\mu\nu}, \phi)$ is the matter field action. In this gravitational system, ϕ^{-2} characterizes the gravitational coupling. Moreover, the scalar field ϕ plays the role of the Higgs field and should therefore have interaction with the matter part of the model so that in the previous action $S_m(g_{\mu\nu}, \phi)$ includes ϕ.

Variation of the action (1) with respect to $g^{\mu\nu}$ and ϕ gives, respectively,

$$\phi^2 G_{\mu\nu} = T_{[m]\mu\nu} + T_{[\phi]\mu\nu}, \tag{2}$$

$$\Box\phi - \phi R + \frac{\partial V}{\partial\phi} = \frac{1}{\sqrt{-g}}\frac{\delta S_m(g_{\mu\nu}, \phi)}{\delta\phi}, \tag{3}$$

where

$$T_{[m]\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m(g_{\mu\nu}, \phi)}{\delta g^{\mu\nu}}, \tag{4}$$

and

$$T_{[\phi]\mu\nu} = -\left(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\nabla_\gamma\phi\nabla^\gamma\phi\right) - \frac{1}{2}\left(g_{\mu\nu}\Box\phi^2 - \nabla_\mu\nabla_\nu\phi^2\right) - g_{\mu\nu}V(\phi). \tag{5}$$

Here $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$ and $G_{\mu\nu}$ is the Einstein tensor. The potential $V(\phi)$ usually contains a quartic self-interaction and an imaginary mass term. This suffices to give minima to the potential for some nonvanishing values of ϕ. If the degenerate vacuum is $\phi = v$ with v being a constant, the spontaneous symmetry breaking then results in a gravitational coupling with the same strength as v^{-2}. This gives the gravitational constant if one takes the mass scale v to be of the same order of the Planck mass. This means that the spontaneous symmetry breaking should take place at Planck scale. In lower energy scales, however, there is no difference between this model and general relativity, since with $\phi = v$ the field equations (2) and (3) reduce to the Einstein field equation, namely,

$$G_{\mu\nu} + v^{-2}V(v)g_{\mu\nu} = v^{-2}T_{[m]\mu\nu}. \tag{6}$$

It is well known that such a symmetry breaking induces a large vacuum energy density in the gravitational equations. This problem is not, however, addressed in the Zee's model, since it is assumed that $V(v) = 0$. In the following, we shall recognize this vacuum energy problem as one of the two main problems affecting the model described by the action (1). The other problem is that this model is not consistent with Mach's principle, since it proposed that gravity is described by general relativity in all energy scales lower than the Planck scale (Brans and Dicke, 1961).

3. THE MODEL

We consider a scalar tensor theory consisting of a real scalar field ϕ, described by the action functional[3]

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\left\{g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{6}\phi^2 R + \lambda\phi^4\right\}, \tag{7}$$

where λ is a dimensionless coupling constant. This action is invariant under conformal transformations

$$\bar{g}_{\mu\nu} = e^{2\sigma}g_{\mu\nu}, \tag{8}$$

$$\bar{\phi} = e^{-\sigma}\phi, \tag{9}$$

where σ is a dimensionless spacetime function. When this action is taken as the vacuum sector of a gravitational model, one encounters an inherent ambiguity concerning the incorporation of matter systems. In fact, since the vacuum sector is conformally invariant, it is not possible to make any distinction between two different conformal frames and all the frames must be considered as dynamically equivalent. In this situation, it is not clear to which of these conformal frames, or the corresponding metric tensors, the matter systems should be coupled. To consider the most general case, we take the matter systems to be coupled with the metric $\bar{g}_{\mu\nu}$ rather than $g_{\mu\nu}$ which are conformally related due to (8). In this way, we take into account all the dynamical implications of different conformal frames. We therefore write the action (7) in the form

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\left\{g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \lambda\phi^4 + \phi^2(g^{\mu\nu}\nabla_\mu\sigma\nabla_\nu\sigma + \frac{1}{6}R)\right\}$$

[3] This action is identical to the gravitational part of (1), if $\frac{1}{2}\phi^2 R$ is replaced by $\frac{1}{12}\phi^2 R$ and $V(\phi)$ is taken to be $-\frac{1}{2}\lambda\phi^4$.

$$+ S_m(\bar{g}_{\mu\nu}, \phi), \tag{10}$$

where $S_m(\bar{g}_{\mu\nu}, \phi)$ is the matter action containing some matter field variables, including the gauge fields of the standard model, coupled to the metric $\bar{g}_{\mu\nu}$. We shall consider the case that the matter action interacts with the scalar field ϕ. This is necessary for interpretation of the scalar field as a Higgs field. We have also taken the previous action to involve a kinetic term for σ to account for its dynamical contributions. In this case, the vacuum sector of the action (10) is still conformally invariant, since σ as a dimensionless function does not change under conformal transformations.

Variation of the action (10) with respect to $g^{\mu\nu}$, ϕ and σ, yields,

$$G_{\mu\nu} - 3\lambda\phi^2 g_{\mu\nu} = 6\phi^{-2}(T_{\mu\nu}(\bar{g}_{\mu\nu}) + \tau_{\mu\nu}) + 6t_{\mu\nu}, \tag{11}$$

$$\Box\phi - \frac{1}{6}R\phi - 2\lambda\phi^3 - \phi\nabla_\gamma\sigma\nabla^\gamma\sigma = -\frac{\delta}{\delta\phi}S_m(\bar{g}_{\mu\nu}, \phi), \tag{12}$$

$$\nabla_\mu(\sqrt{-g}\phi^2 g^{\mu\nu}\nabla_\nu\sigma) = \sqrt{-g}g^{\mu\nu}T_{\mu\nu}(\bar{g}_{\mu\nu}), \tag{13}$$

where

$$T_{\mu\nu}(\bar{g}_{\mu\nu}) = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}S_m(\bar{g}_{\mu\nu}, \phi), \tag{14}$$

and

$$\tau_{\mu\nu} = -(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\nabla_\gamma\phi\nabla^\gamma\phi) - \frac{1}{6}(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\phi^2, \tag{15}$$

$$t_{\mu\nu} = -(\nabla_\mu\sigma\nabla_\nu\sigma - \frac{1}{2}g_{\mu\nu}\nabla_\gamma\sigma\nabla^\gamma\sigma). \tag{16}$$

We would like to consider breakdown of the conformal invariance in the action (10). Therefore, we add a term such as

$$-\frac{1}{2}\int d^4x\sqrt{-g}m^2\phi^2, \tag{17}$$

to this action with m being a constant mass scale. In this case, Equations (11) and (12) change to

$$G_{\mu\nu} - 3(m^2 + \lambda\phi^2)g_{\mu\nu} = 6\phi^{-2}(T_{\mu\nu}(\bar{g}_{\mu\nu}) + \tau_{\mu\nu}) + 6t_{\mu\nu}, \tag{18}$$

$$\Box\phi - \frac{1}{6}R\phi - m^2\phi - 2\lambda\phi^3 - \phi\nabla_\gamma\sigma\nabla^\gamma\sigma = -\frac{\delta}{\delta\phi}S_m(\bar{g}_{\mu\nu}, \phi), \tag{19}$$

Looking at the gravitational equation (18) one infers from the sign of the mass term that it has a negative contribution to vacuum energy density. This feature seems to be generic to all scalar tensor theories (namely theories that consider a scalar field with nonminimal coupling to gravity) which entail a massive scalar field. The sign of this mass term can however be changed by introducing a constant mass scale such as μ with $\mu^2 = -m^2$. It is important that this induces spontaneous symmetry breaking in the action (10).[4] In this case, the potential of the scalar field can be written as $U(\phi) = -\mu^2\phi^2 + \lambda\phi^4$. The minimum of this potential is determined by the conditions

$$\frac{dU}{d\phi} = -2\mu^2\phi + 4\lambda\phi^3 = 0, \tag{20}$$

and

$$\frac{d^2U}{d\phi^2} = -2\mu^2 + 12\lambda\phi^2 > 0, \tag{21}$$

where $\lambda > 0$. The relation (20) has nonzero solutions $\phi_0^2 = \frac{\mu^2}{2\lambda}$ minimizing the potential at $U(\phi_0) = \frac{-\mu^4}{4\lambda}$. The gravitational coupling is then given by $\phi_0^{-2} \sim \lambda\mu^{-2}$. This is the gravitational constant if $\mu \sim \lambda^{\frac{1}{2}} m_{\rm p}$ with $m_{\rm p} \sim G^{-\frac{1}{2}}$ being the Planck mass. Note that the energy scale at which the spontaneous symmetry breaking takes place is not necessarily the Planck scale. It is given by μ which depends on the precise value of the coupling constant λ.

For $\phi = \phi_0$, the action (10), together with (17) and $\mu^2 = -m^2$, reduces to

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\{R - 2\Lambda + 6g^{\mu\nu}\nabla_\mu\sigma\nabla_\nu\sigma\} + S_m(\bar{g}_{\mu\nu}), \tag{22}$$

where $\Lambda = \frac{3}{2}\mu^2$. It is clear that the result of the spontaneous symmetry breaking in (10), namely the action (22), is very different from that obtained by the action (1). In the action (22), the metric tensors in the gravitational and the matter parts belong to different conformal frames and the conformal factor itself appears as a dynamical field. The consequences of such a coupling are discussed in the next section.[5]

4. MACH'S PRINCIPLE AND THE COSMOLOGICAL CONSTANT

When a constant mass scale such as m^2 (or $-\mu^2$) is introduced into the gravitational part of the action (10), the conformal invariance is broken and a particular conformal frame (or unit system) is singled out in terms of the values attributed to the dimensional quantities. In general, the use of two different unit systems are conventional. On one hand, the gravitational constant as a dimensional coupling characterizes a unit system usually used in cosmology which is referred to as the cosmological frame (Bisabr and Salehi, 2002). On the other hand, there is a particle unit system used in elementary particle physics and is defined in

[4] Note that μ appears as a tachyonic mass in the field equation (19).

[5] The coupling of matter systems with a metric which is conformally related to the background metric tensor is previously used in different contexts, see for example, Cho (1992), and Damour *et al.* (1990).

terms of Compton wavelength of a typical elementary particle. It is important to note that one usually assumes that these two unit systems are related by a global unit transformation. This means that they are taken to be indistinguishable up to a constant conversion factor in all spacetime points. Such a unit transformation is obviously devoid of any dynamical implication. Here we consider a local unit transformation (Bekenstein and Meisels, 1980; Dicke, 1962) which requires that the two different unit systems be interrelated by a spacetime dependent conversion (or conformal) factor. This gives a dynamical meaning to changes of unit systems and seems to be more consistent with Mach's principle. The reason for this is discussed in the following paragraphs.

According to Mach's principle, one may attribute inertial forces experienced in a given point of spacetime to the gravitational forces due to distant accelerated matter systems. By implication, the inertial mass of a particle should depend on the distribution of matter around that particle. As a consequence, one expects that inertial masses have different values in different spacetime points.

On the other hand, there is an inherent ambiguity concerning measurement of changes of a dimensional quantity. In general, the value of a dimensional quantity not only may change in a given unit system but it may also change due to changes of the unit system by which the quantity is measured. There is not however any direct way to distinguish between these two types of changes. Thus, it is only meaningful to compare mass ratios, as dimensionless quantities, rather than mass itself at different spacetime points. For construction of such a dimensionless quantity one may take proportion of the inertial mass of a typical elementary particle, M, and the Planck mass, namely,

$$M(G)^{1/2} = n, \tag{23}$$

where n is a dimensionless number. If one takes n to vary, as suggested by Mach's principle, one may consider two possibilities depending on attribution of these variations to either M or G. In the particle frame, one takes M as a constant and G as varying, while in the cosmological frame inertial masses of elementary particles are taken to be varying and G is regarded as a constant. It is now clear from the discussion that these two unit systems should be taken to be related by a spacetime dependent conversion factor in order to respect the precise statement of Mach's principle. It should be remarked that from a physical point of view there seems to be no fundamental difference between the two possibilities, although their precise formulations may need theories which have quite different mathematical structures. In the present work, we shall not concern with this issue and only the physical content of the dynamical distinction of the two unit systems is brought into focus.

Now we turn to interpret the action (22). For doing this, we recall that when the action (1) undergoes spontaneous symmetry breaking both the gravitational coupling and the inertial masses have constant configurations contrary to the statement of Mach's principle. On the other hand, the appearance of σ as a dynamical field in the action (22) implies that inertial masses of elementary particles have variable configurations even though the gravitational coupling takes a constant value.[6] This is due to the fact that matter is coupled to gravity through the metric $\bar{g}_{\mu\nu}$ which is conformally related to $g_{\mu\nu}$. We emphasize that this feature is a direct consequence of the conformal invariance of the vacuum sector of the action (10) which gives plausibility to consider such a coupling.

As the last point, we investigate the appearance of Λ in (22) as a large effective vacuum energy density. We remark that this does not lead to the cosmological constant problem in the context of our model which assumes that the cosmological and the particle unit systems are dynamically distinct. To clarify this point we first take the two metric tensors $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ to describe the cosmological and the particle frames, respectively. They are related by $g_{\mu\nu} = e^{-2\sigma}\bar{g}_{\mu\nu}$. Such a distinction should also be imposed on the dimensional quantities Λ and $\bar{\Lambda}$, namely the value of vacuum energy density in the cosmological and the particle unit systems. According to the dimension of Λ (the squared mass) this two quantities are related by $\Lambda = e^{2\sigma}\bar{\Lambda}$. Thus, Λ is not actually a constant in the cosmological frame and the action (22) should be written as

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\{R - 2\bar{\Lambda}e^{2\sigma} + 6g^{\mu\nu}\nabla_\mu\sigma\nabla_\nu\sigma\} + S_m(\bar{g}_{\mu\nu}), \tag{24}$$

which leads to the field equations

$$G_{\mu\nu} + \bar{\Lambda}e^{2\sigma}g_{\mu\nu} = 8\pi G T_{\mu\nu}(\bar{g}_{\mu\nu}) + 6t_{\mu\nu}, \tag{25}$$

$$\Box\sigma + \frac{1}{3}\bar{\Lambda}e^{2\sigma} = \frac{4\pi}{3}Gg^{\mu\nu}T_{\mu\nu}(\bar{g}_{\mu\nu}). \tag{26}$$

In these equations the exponential coefficient for $\bar{\Lambda}$ emphasizes the dynamical distinction between the cosmological and the particle unit systems. One intuitively expects that this distinction be indistinguishable immediately after the spontaneous symmetry breaking. This means that $g_{\mu\nu}$ and Λ coincide with their corresponding quantities in the particle unit system, namely $\bar{g}_{\mu\nu}$ and $\bar{\Lambda}$, at sufficiently early times. When the universe expands, cosmological scales enlarge and this distinction increases so that $e^{-2\sigma}$ must be an increasing function of time in an expanding universe. It follows that Λ characterizing the effective cosmological constant in the cosmological frame damps due to the cosmic expansion (Bisabr and Salehi, 2002).

[6] It is important to note that when the conformal invariance of the gravitational part of (10) is broken the resulting theory automatically chooses the cosmological frame. This is a direct consequence of the spontaneous symmetry breaking in the action (10) that gives a constant configuration to the scalar field.

As an illustration, we first combine (26) with the trace of (25) that gives

$$\Box\sigma + \nabla_\gamma \sigma \nabla^\gamma \sigma + \frac{1}{6} R - \frac{1}{3}\bar{\Lambda} e^{2\sigma} = 0. \tag{27}$$

If we write this equation in a spatially flat Friedmann–Robertson–Walker space-time, we obtain[7]

$$\ddot{\sigma} + 3\frac{\dot{a}}{a}\dot{\sigma} + \dot{\sigma}^2 - \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} + \frac{1}{3}\bar{\Lambda} e^{2\sigma} = 0, \tag{28}$$

where $a(t)$ is the scale factor and the overdot denotes differentiation with respect to time.

Now assuming that the universe follows a power-law expansion, namely that $\frac{\dot{a}}{a} \sim t^{-1}$, we use the ansatz

$$e^{-\sigma} = \sigma_0 t, \tag{29}$$

which satisfies Equation (28) with $\sigma_0 \sim \sqrt{\bar{\Lambda}}$. The vacuum energy density in the cosmological frame is then $\Lambda = \bar{\Lambda} e^{2\sigma} \sim t^{-2}$, which is consistent with the observational upper limit.

5. CONCLUDING REMARKS

We have investigated the idea that gravitational coupling may be the result of a mechanism involving the breakdown of some fundamental symmetry of nature. Two kinds of symmetry breaking seem to be relevant on the subject: firstly, it is natural to think about spontaneous symmetry breaking analogous to the origin of coupling constants corresponding to other fundamental interactions and secondly, a conformal symmetry breaking motivated by the fact that the gravitational coupling is a dimensional coupling.

We discussed spontaneous symmetry breaking in a conformally invariant scalar tensor theory, in which the scalar field has a quartic self-interaction, by introducing a constant tachyonic mass scale. We have shown that a spontaneous symmetry breaking in the model would lead to a gravitational coupling as suggested by general relativity. It should be remarked that this symmetry breaking may take place in energy scales much lower than the Planck scale when $\lambda << 1$.

We emphasize that our analysis allows to avoid the two problems afflict the model proposed by Zee, the action (1), namely inconsistency with Mach's principle and the cosmological constant problem. This is basically due to the fact that in the action [**?**] we couple the matter to a metric which is conformally related to that describing the gravitational part. In our approach, such a coupling is motivated by the fact that the gravitational part is conformally invariant and

[7] Due to homogeneity and isotropy of the universe σ is taken to be only a function of time.

does not dynamically distinguish between different metric tensors related by the conformal transformation (8). The important feature of this coupling is that it gives variable configurations to all the mass scales introduced by elementary particle physics in the cosmological frame. In particular, contributions of these mass scales in vacuum energy density are so that they decrease during evolution of the universe.

We point out that the nontrivial behavior of the conformal factor in our model may have important role in cosmology. For instance, it may act as a quintessence describing the acceleration of the universe in the present epoch (Bisabr, 2004).

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